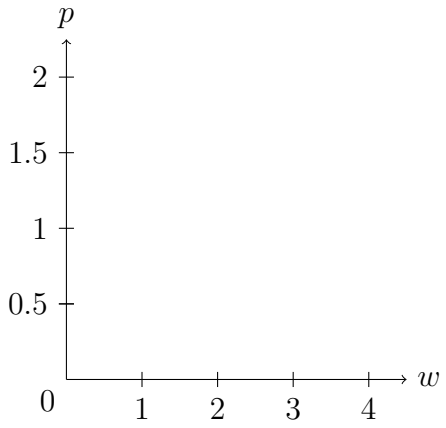
1. **People still use snail-mail?** First-class postage is currently \$0.78 for a letter weighing up to and including 1 oz. with \$0.29 for each additional ounce (up to and including each additional ounce).

   (a) Graph the function $p = f(w)$ that gives the postage p for sending a letter weighing w ounces for $0 < w \leq 3.5$.



   (b) Evaluate $\lim_{w \to 2.3} f(w)$.

   (c) Does $\lim_{w \to 3} f(w)$ exist? Explain.

   (d) Write a formula for $f(w)$.

2. **General Relativity** Suppose a spaceship of length L_0 travels at a high speed v relative to an observer. To the observer, the ship appears to have a length L smaller than L_0 given by the *Lorentz contraction formula*:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

where c is the speed of light.

(a) What is the observed length L of the ship if it is traveling 50% the speed of light?

(b) What is the observed length L of the ship if it is traveling 75% the speed of light?

(c) What happens to L as the speed of the ship increases?

(d) Find and interpret

$$\lim_{v \to c^-} L_0\sqrt{1 - \frac{v^2}{c^2}}.$$

3. **Difference of Powers** There is a convenient factorization formula of the form

$$x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \cdots + a^{n-3}x^2 + a^{n-2}x + a^{n-1})$$

Use this formula to evaluate

(a) $\lim_{x\to 5} \dfrac{x^3 - 125}{x - 5}$

(b) $\lim_{x\to 1} \dfrac{x^6 - 1}{x - 1}$

(c) $\lim_{x\to -1} \dfrac{x^7 + 1}{x + 1}$

(d) $\lim_{x\to a} \dfrac{x^5 - a^5}{x - a}$

(e) $\lim_{x \to a} \dfrac{x^n - a^n}{x - a}$

4. **Breaking the Limit Laws?** Find functions $f(x)$ and $g(x)$ such that $\lim_{x \to 1} f(x) = 0$ and

$$\lim_{x \to 1} \big(f(x)g(x)\big) = 5$$

5. **Smoothing sin $\frac{1}{x}$.** Show that $-|x| \leq x \sin \frac{1}{x} \leq |x|$.

Use the above to find $\lim_{x \to 0} x \sin \dfrac{1}{x}$.